# Experimental observation of the group velocity of thermal waves propagating in various materials

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**Abstract**

This study investigates the propagation velocity of thermal waves, which is fundamental knowledge for non-destructive testing using active thermography techniques. We believe that, although the conventionally considered thermal wave velocity is phase velocity, the thermal energy should propagate at the group velocity. The results of theoretical calculations estimated that the group velocity of thermal waves was twice the phase velocity. Experiments were performed on aluminium and stainless steel specimens, and the results showed that the measured group velocities agreed with the theoretical estimates in consideration of propagating and complex attenuations.

## 1. Introduction

Propagation velocity of thermal waves is a fundamental knowledge for designing active thermography non-destructive testing (such as lock-in thermography technique). It has been discussed in the literatures [1,2] and obtained from the heat conduction equation assuming a boundary condition of periodic temperature change as:

$$v_p = \frac{\omega}{k} = \frac{\omega}{\sqrt{\omega/2\alpha}} = \sqrt{2\alpha\omega} = 2\sqrt{\pi f\alpha} \qquad (1)$$

where $\omega$, $k$, and $\alpha$ denote angular frequency (= $2\pi f$), wave number, and thermal diffusivity, respectively. In Eq. (1), $k = \sqrt{\omega/2\alpha}$ is derived from the solution to the heat conduction equation. However, the velocity obtained by Eq. (1) is considered the phase velocity in wave theory; whereas, in the wave theory, wave energy (or wave packets) propagates at the group velocity. In this study, the group velocity of thermal waves is estimated based on the theory of group velocity, and the result is verified through experiments on aluminium and stainless steel specimen.

## 2. Estimation of thermal wave group velocity

From Eq. (1), $v_p$ can be expressed as $v_p = 2\alpha k$. Therefore, the group velocity of thermal waves $v_g$ is estimated based on group velocity theory and the $v_p$ as:

$$v_g = \frac{d\omega}{dk} = \frac{d}{dk}(kv_p) = v_p + k\frac{dv_p}{dk} = 4\alpha k = 2v_p \qquad (2)$$

Equation (2) shows that the group velocity of thermal waves should be twice its phase velocity.

## 3. Experiment setup

Two square bar-shaped specimens (aluminium and stainless steel specimens, 5 mm × 5 mm ×1000 mm) were used in the experiments. One end of each specimen was periodically heated to excite thermal waves using a halogen lamp whose output power was controlled by a function generator (MFG-2230M, TEXIO). The lamp voltage was modulated as a five-cycle sine waves; the modulation frequency was 0.005–0.04 Hz. The temperatures on the side surface of the specimens during the heating were monitored by an infrared camera (A315, FLIR). Then, temperature–time data at ten different points along the length of the specimen were obtained, and the thermal wave velocities were obtained from the propagation time delay of the excited thermal wave packets.

## 4. Result

Figure 1(a) shows an example of observed thermal image (results on aluminium specimen when excitation frequency was 0.005 Hz), and the temperature–time data obtained from the data in Figure 1(a) is shown in Figure 1(b). When the propagation velocity of thermal waves is calculated based on the time delay of the local maximum peak values, the resulting velocity matches the phase velocity $v_p$ estimated from Eq. (1). In order to obtain the group velocity $v_g$, the temperature–time data were bandpass filtered, and the propagation of envelopes of the five-cycle data was focused on. Figure 2(a) shows the envelopes of the temperature–time data, and Figure 2(b) shows the relationship between the time delay of the envelopes and the measurement distance from the heating point; thus, the slope of the data in Figure 2(b) indicates the propagation velocity. Figure 3 compares the velocities obtained at each excitation frequency with the theoretically estimated group velocity (calculated using Eq. (2)). The results for the aluminium specimen (Figure 3(a)) agreed well with the theoretical estimation ($\alpha$ = 98.8 mm²/s). This indicates that the obtained velocities are group velocities, thereby verifying the theoretical calculation (Eq. (3)). On the other hand, the results for stainless steel specimen (Figure 3(b)) were smaller than the theoretical estimation ($\alpha$ = 3.49 mm²/s). This should be because of large attenuation of thermal wave in the steel. When discussing the wave propagation in the materials with significant attenuation, consideration of complex phase velocity $v'_p$ and complex group velocity $v'_g$ (= 2 $v'_p$) is necessary. The complex velocities contain an imaginary part that represents the attenuation coefficient, and those values should be smaller than $v_p$ and $v_g$. The complex group velocity $v'_g$ of stainless steel was determined by measuring its thermal wave attenuation coefficient, and the results are presented in Figure 3(b) as a dashed line. It is found from Figure 3(b) that the experimentally obtained velocities agree with the complex group velocities.

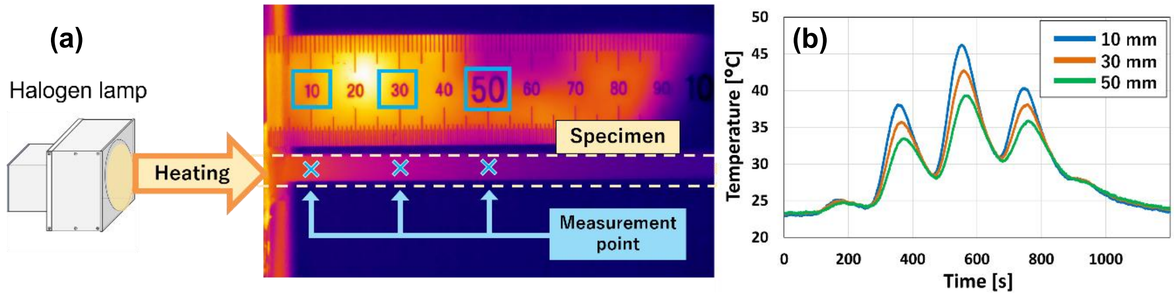


**Figure 1.** (a) Experimentally observed thermal image (showing side surface of specimen), (b) Temperature–time data at three different measurement points (10, 30, and 50 mm from heating surface).

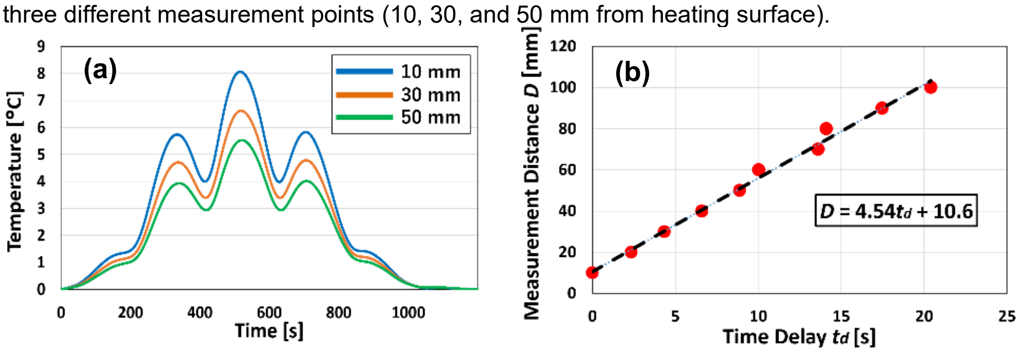


**Figure 2.** (a) Envelopes of temperature–time data, (b) Relationship between measurement distance and time delay of envelopes (slope indicates propagation velocity of envelopes).

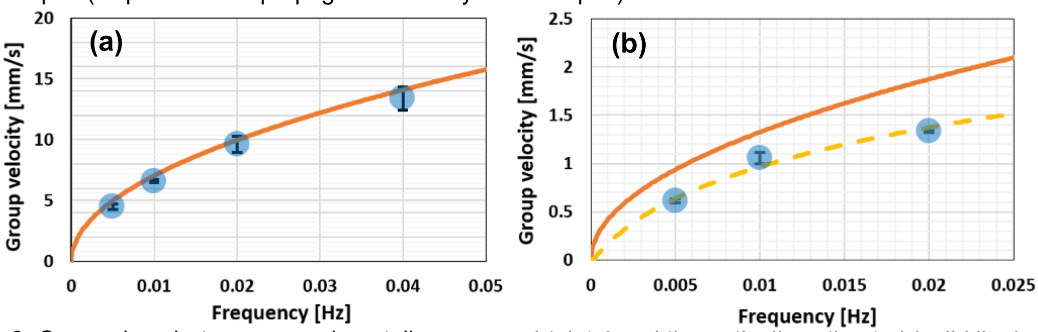


**Figure 3.** Comparison between experimentally measured (plots) and theoretically estimated (solid line) group velocities of (a) aluminium, and (b) stainless steel specimen. Dashed line in (b) shows complex group velocity.

## 5. Conclusion

The group velocity of thermal waves was theoretically estimated, and the result was verified through experiments. The experimental results agreed with the theoretical predictions and indicated that thermal waves (or wave packets) propagate at the group velocity, which is twice the phase velocity. These findings provide fundamental information for determining appropriate testing durations in active thermography.

### References

[1] D.P. Almond, P.M. Patel, «Photothermal Science and Techniques», Chapman and Hall, London, 1996.

[2] X. Maldague, «Theory and practice of infrared technology for nondestructive testing», Wiley, 2001.