Bayesian inference for the simultaneous estimation of thermal properties of orthotropic material

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Extended abstract

Determining the components of thermal conductivity of orthotropic or anisotropic materials is the subject of much experimental and theoretical researches. A characteristic of these characterization experiments is that the observable is often unique, for example, the temperature of the rear face when the front face is thermally stimulated. Therefore, only partial information is available regarding the spatial distribution of the flux. The simultaneous visualization, using a single camera (with optics), of both the front and back faces of a plate subjected to laser heating was recently proposed by Houssein et al. to estimate the thermal diffusivity of liquid metals [1]. While the experimental setup is more complex in terms of optical path length, two advantages can be highlighted. First, a single camera simplifies calibration and synchronization, and second, the observable and the input are of the same nature. On the other hand, a temperature-temperature model makes possible to express a transmittance that depends solely on diffusivity and rear face losses, without considering the absorbed portion of the laser flux or front face losses. Therefore, it reduces the number of unknowns.

The initial developments toward using a single infrared camera to simultaneously observe four surfaces of an orthotropic solid are presented with the aim of visualizing where the flux goes when one of the surfaces is subjected to laser heating (Figure 1). The experimental setup consists of nine mirrors and two prisms, which allow the infrared camera to simultaneously view four of the six faces of the parallelepiped (Figure 2). The four optical paths are identical to ensure clear images. The sample is a rectangular wooden (balsa) parallelepiped with a square base of $4\ell^2$ and a thickness of e. The laser heating must be adjusted so that the sample temperature remains below 10 K at its maximum.

The main objective is to estimate the three components of the thermal conductivity of the parallelepiped-shaped orthotropic solid. The observations taken with the infrared camera on the heated surface are used as a boundary condition, while the measurements taken on three other surfaces are used in the functional for solving an inverse problem.

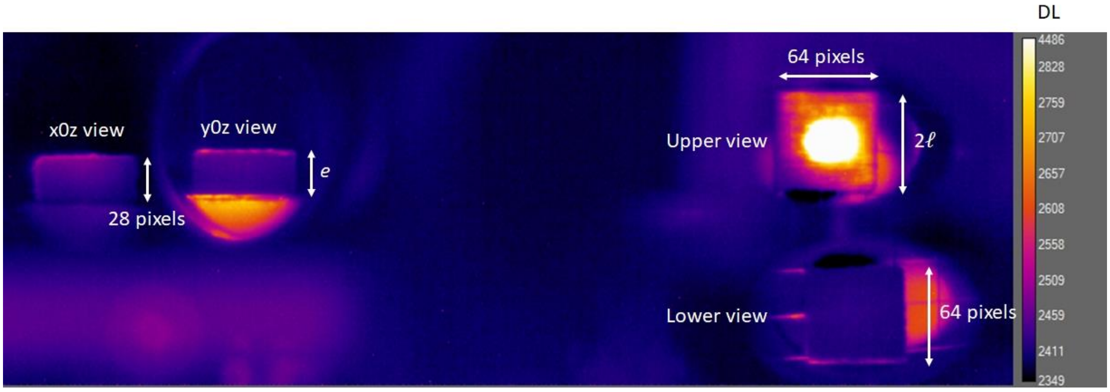

Figure 1. Simultaneous infrared image of the four faces (scale in DL).

A Fourier-type integral transform is applied to the measurements used for the inverse analysis, using the eigenvalues related to the original thermal conduction problem [2-3] (Figure 3). Few transformed modes are used to solve the inverse problem, thus spatially compressing the data provided by the infrared camera [2,4]. The inverse problem is solved within the Bayesian framework of statistics, by applying the Metropolis-Hastings algorithm with sequential sampling in two sets of model parameters (parameters of interest and others) [5].

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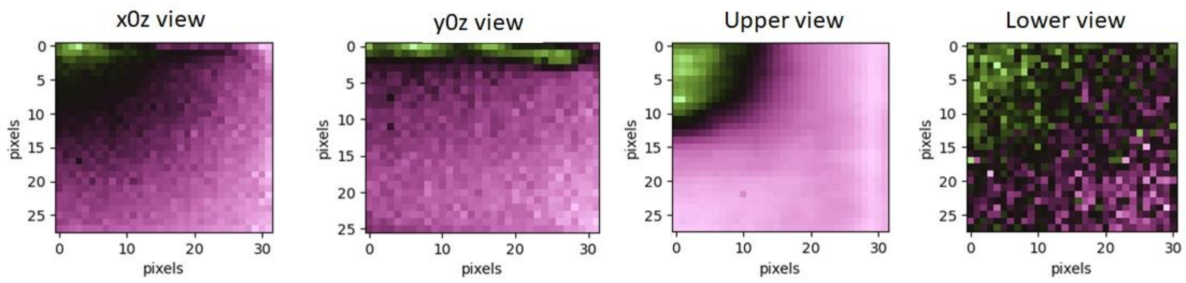

Figure 2. Temperature fields for the four faces (the scale is not the same from one face to the other).

Experimentally, the upper face is heated by a Gaussian type laser heater. However, for the estimation, within the framework of this first theoretical approach, it is a hypothetical homogeneous response in temperature at z = 0 which is considered as input data. And synthetic data are obtained through a simulation performed with Comsol multiphysics®. Since the upper face have a homogeneous temperature, the estimation results of the conductivities and heat capacity are consistent but unequal error values compared to those of the direct model, even if the temperature evolution over time are satisfying (Figures 3 and 4).

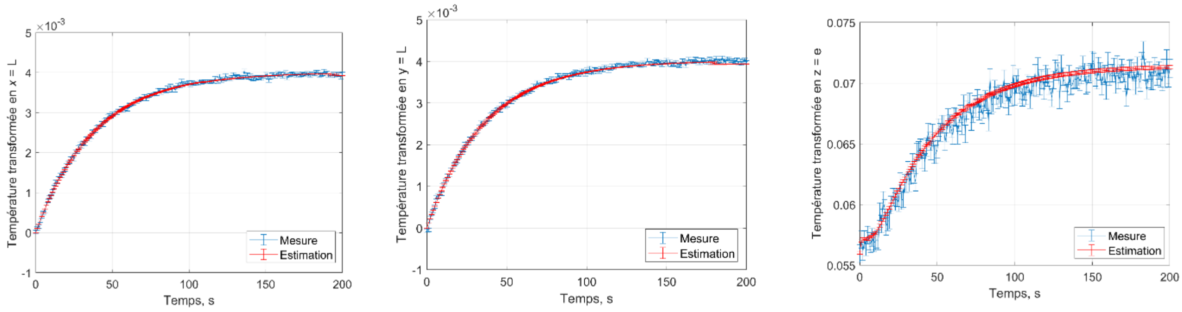

Figure 3. Estimated and simulated transformed temperatures in $x = \ell$, in $y = \ell$, and $z = e$.

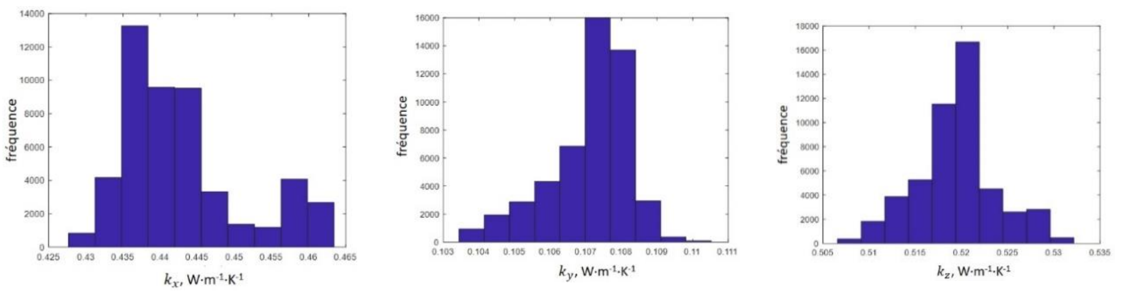

Figure 4. Histogram of the estimated conductivities k_x, k_y, and k_z.

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